

# Statistical challenges in the study of aging and dementia

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# Some challenges for statistics...

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- **Change** is key
- **Measurement** is tough
  - Errors
  - Multidimensional constructs
  - Indirect; maybe differential
- Sicker people refuse, **drop out**, skip, die
- Aging is **complex**



**biases**

# ...leads to statistical challenges

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- *Longitudinal* data analysis
- *Measurement* modeling
  - Errors-in-variables
  - Data reduction techniques, e.g., principal components
  - Latent variable modeling
- *Missing data, competing risks* analysis
- Mathematical *modeling*

# Objective

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*For you to walk away  
with **specific**, **useful**  
information on at least  
one of the challenges:*

# ...leads to statistical challenges

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- *Longitudinal* data analysis
- *Measurement* modeling
  - Errors-in-variables
  - Data reduction techniques, e.g., principal components
  - Latent variable modeling
- *Missing data, competing risks* analysis
- Mathematical *modeling*

# Why longitudinal data analysis (LDA)?

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- Top ten reasons

10. Because it will make me look so cool

9. Because a grant reviewer will call my application “unsophisticated” if not

*(I'm only creative enough to come up with two of these....)*

# Why LDA?

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- Top four reasons

4. To inform policy

- Changes in disability prevalence over time

3. To study natural histories

- Functional trajectories and their etiologies

2. To make prognoses, incorporating history

- Cognitive status transitions

1. To progress from “association” toward “cause”

- Intervention A or risk adoption B changes outcomes

# What I Hope You'll Get Out of This

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- The basic longitudinal modeling methods
- How one implements those methods
  - Key models
  - Software
- Heads up on the primary challenges



# An Example

## Emotional vitality and mobility

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- Study: Women's Health & Aging ( $n=1002$ ; Guralnik et al., 1995)
- Question: Does emotional vitality affect mobility trajectory?
  - Emotional vitality (X: 1 if vital; 0 ow)
    - High mastery, being happy, few depressive/anxious symptoms
  - Penninx et al., 2000*
  - Mobility (Y)
    - Usual walking speed (max 2 trials)
    - Indicator of severe walking difficulty (1 if yes; 0 ow)
  - Time (T)
    - Study rounds 0-6

# The basic longitudinal methods

*Diggle, Heagerty, Liang & Zeger, 2001*

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- Top four reasons

4. To inform policy

- Population average (marginal models; GEE)

3. To study natural histories

- Subject-specific (random effects; growth curves)

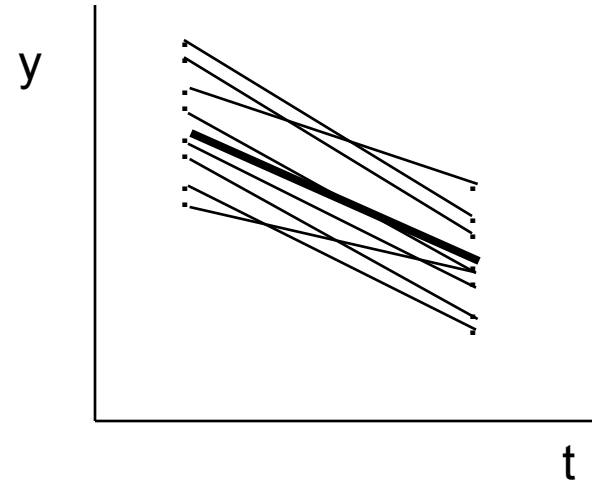
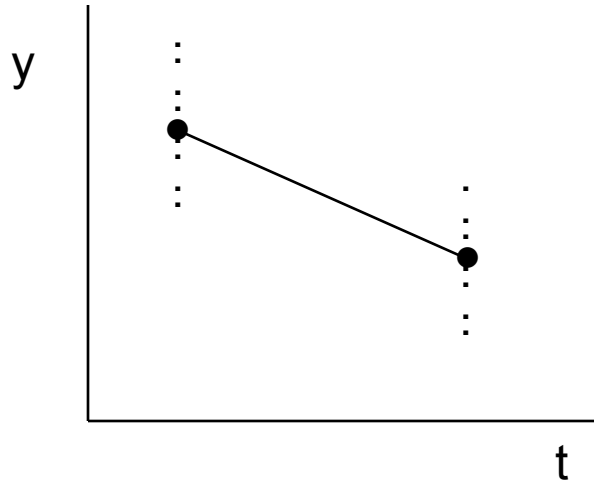
2. To make prognoses, incorporating history

- Transitions (autoregressive & Markov models)

1. To progress from “association” toward “cause”

- Time-varying covariates (with complexities)

# Population average v. Subject-Specific



- PA: Compare populations over time
  - (Fixed) time effect = **slope of the averages**
- SS: Compare women **to selves** over time
  - (Fixed) time effect = **average of the slopes**
- Subtle point: These are **equal**
  - with **continuous** outcomes Y (linear regression); **NOT otherwise**
  - provided that **within-person correlation** is explicitly **accounted for**

# Population-average models

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- Keywords
  - Marginal models
  - GEE (Generalized Estimating Equations)  
*Liang & Zeger, 1986*
  - Panel analysis
- Sound bites
  - Focus usually on averages (their trajectories)
  - Serial correlation often a “nuisance”
  - “Robust”

# Population-average models

## Description of average trajectories

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- Model—time-invariant covariates:

$$Y_{i1} = \beta_0 + \beta_1 x_i + \beta_2 t_{i1} + \beta_3 x_i \cdot t_{i1} + e_{i1}$$

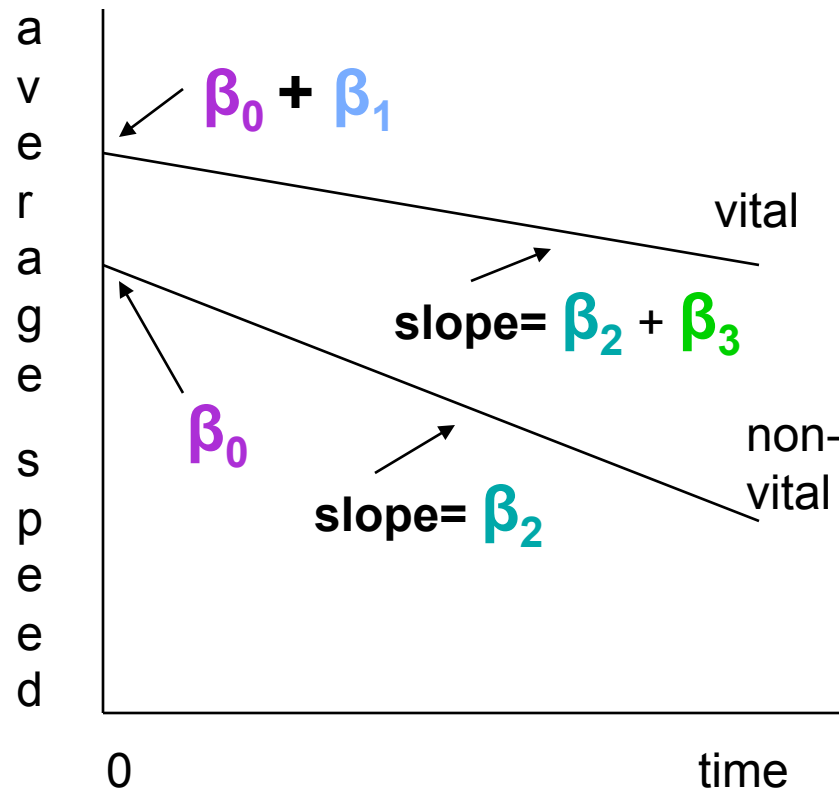
$$Y_{ij} = \beta_0 + \beta_1 x_i + \beta_2 t_{ij} + \beta_3 x_i \cdot t_{ij} + e_{ij}$$

$$Y_{i7} = \beta_0 + \beta_1 x_i + \boxed{\beta_2} t_{i7} + \boxed{\beta_3} x_i \cdot t_{i7} + e_{i7}$$

- Key points**
    - rate of change in average walk speed of non-vital persons
    - Greek = “fixed”; Roman = variable
    - “ANCOVA” model
    - **Coding: main effects for “treatment,” time; interaction**
    - Note contrast viz “change scores”: more powerful
- amount rate of change in average walk speed differs between vital & non-vital persons

# Population-average models

## Pictures



- Data displays
  - Side-by-side box plots (by time, “treatment”)
  - Connect-the-means plots (over time, by treatment)
  - Y versus t smoothed scatterplot, per x

$$Y_{ij} = \beta_0 + \beta_1 x_i + \beta_2 t_{ij} + \beta_3 x_i \cdot t_{ij} + e_{ij}$$

# Population-average models

## Treatment of serial correlation

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$$Y_{i1} = \beta_0 + \beta_1 x_i + \beta_2 t_{i1} + \beta_3 x_i \cdot t_{i1} + e_{i1}$$

$$Y_{ij} = \beta_0 + \beta_1 x_i + \beta_2 t_{ij} + \beta_3 x_i \cdot t_{ij} + e_{ij}$$

$$Y_{i7} = \beta_0 + \beta_1 x_i + \beta_2 t_{i7} + \beta_3 x_i \cdot t_{i7} + e_{i7}$$

- Key points

- Errors are **correlated within persons**
- Most software: you **choose** the correlation “structure”
  - “**Exchangeable**” – all measures equally strongly correlated
  - “**Autoregressive**,” “**banded**” – measures closer in time more strongly correlated
  - “**Unstructured**” – as it sounds (here: 7 choose 2 = 21 ps)
  - “**Independence**” – all correlations assumed = 0

error: amount that  
speed of woman “i”  
differs from population  
average at time 7

# Population-average models: Fitting

- Software
  - SAS: GENMOD (GEE); MIXED, repeated (MLE)
  - SPSS: Advanced model package
  - Stata: xtgee (GEE); xtreg (MLE)
- GEE versus MLE (maximum likelihood est.)
  - Both: accurate coefficient estimates whether or not correlation structure choice is correct
  - GEE: standard errors also accurate, regardless
  - MLE: More valid handling of missing data



# Subject-specific models

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- Keywords
  - Mixed effects, growth curves, multi-level
  - Mixed model; hierarchical (linear) model GEE  
*Laird & Ware, 1982; Raudenbush & Bryk, 1986*
  - Random coefficient model
- Sound bites
  - Focus usually on individual trajectories
  - “Heterogeneity”: variability of trajectories
  - Assumptions are made, and may matter

# Subject-specific models

## Average & individual trajectories

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- Model—time-invariant covariates:

$$Y_{i1} = \beta_0 + b_{0i} + \beta_1 x_i + \beta_2 t_{i1} + b_{2i} t_{i1} + \beta_3 x_i \cdot t_{i1} + e_{i1}$$

$$Y_{ij} = \beta_0 + b_{0i} + \beta_1 x_i + \beta_2 t_{ij} + b_{2i} t_{ij} + \beta_3 x_i \cdot t_{ij} + e_{ij}$$

$$Y_{i7} = \beta_0 + b_{0i} + \beta_1 x_i + \beta_2 t_{i7} + b_{2i} t_{i7} + \beta_3 x_i \cdot t_{i7} + e_{i7}$$

- Key points:

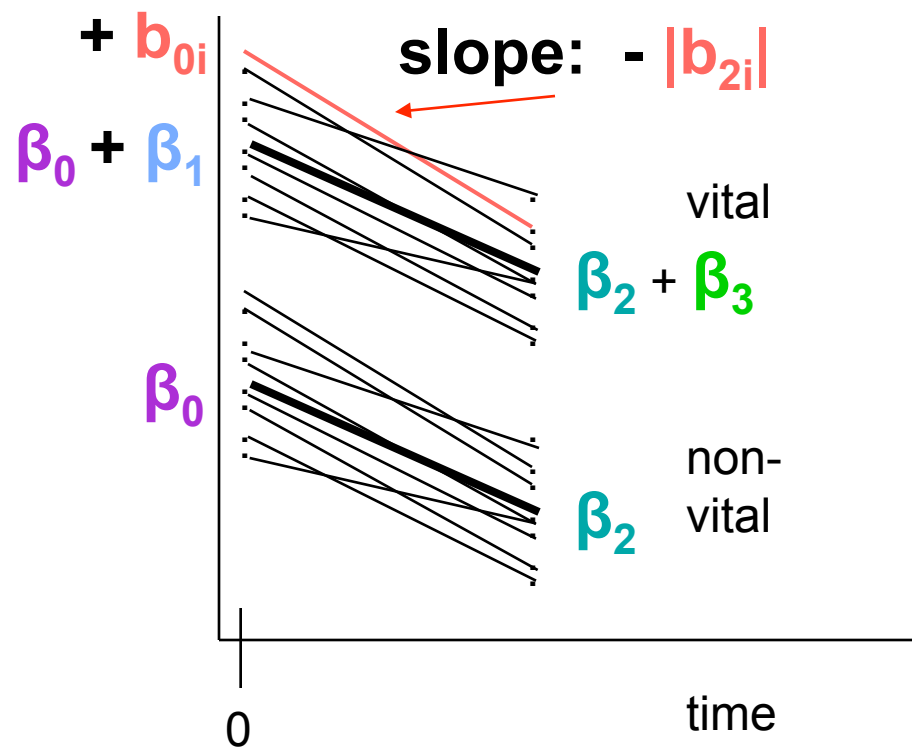
- The additional coefficients are random
- Modeling assumes a distribution: usually normal
  - Distribution variance characterizes “heterogeneity”
  - Heterogeneity results in within-person correlation
- One may define correlation structure for  $e_{ij}$ s too

amount of time  
speed for person i  
exceeds or falls  
short of the average

amount speed  
trajectory for person i  
differs from average

# Subject-specific models

## Pictures



- $b_{0i}$  = random intercept  
 $b_{2i}$  = random slope  
 (could define more)
- heterogeneity  $\longleftrightarrow$   
 spread in intercepts, slopes
- Sentinel data display:  
 spaghetti plot  
*(Ferrucci et al., 1996)*

$$Y_{ij} = \beta_0 + b_{0i} + \beta_1 x_i + \beta_2 t_{ij} + b_{2i} t_{ij} + \beta_3 x_i \cdot t_{ij} + e_{ij}$$

# Subject-specific models: Fitting

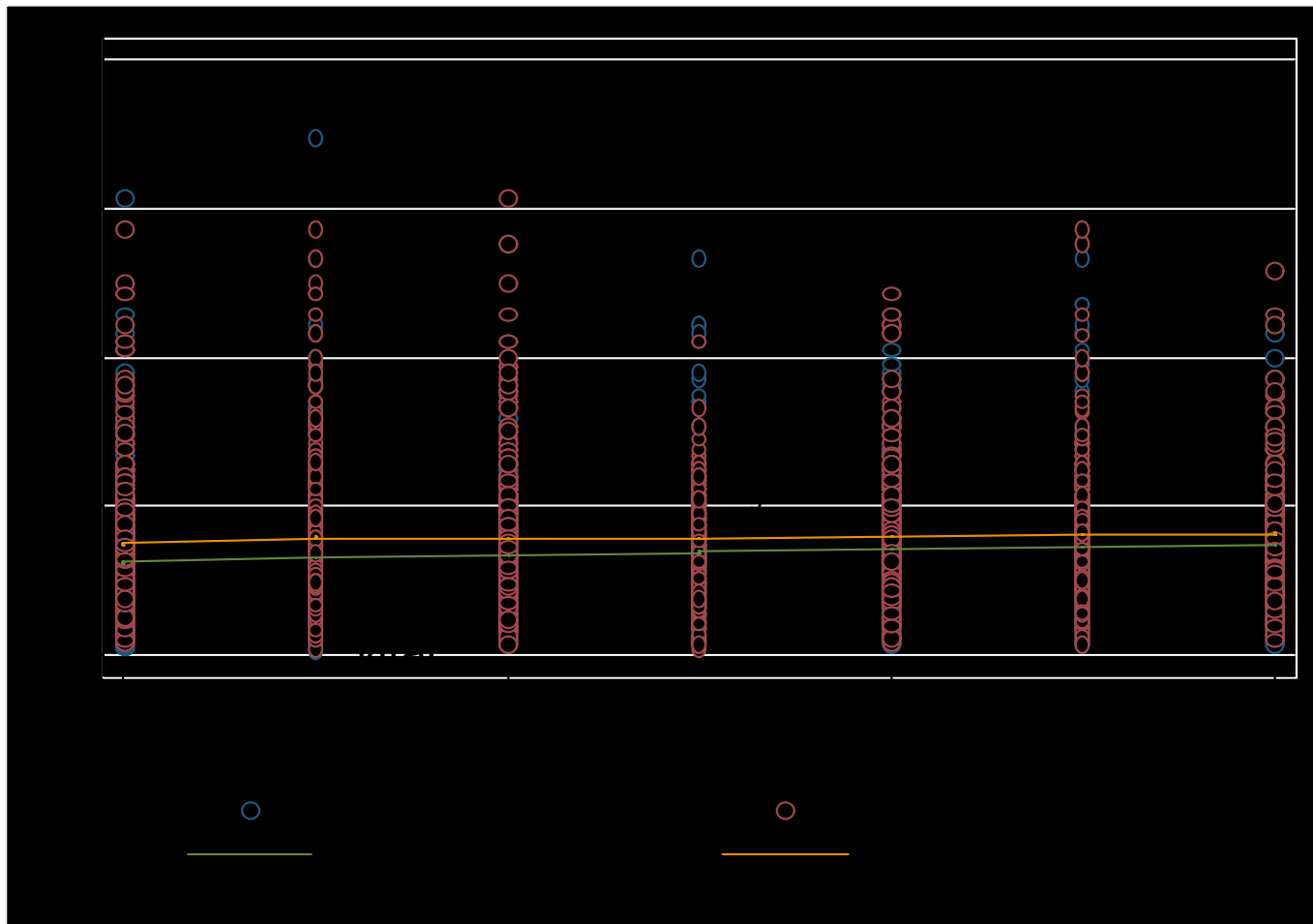
- Software
  - SAS: MIXED, random; GLIMMIX (macro);  
NLMIXED
  - SPSS: Advanced model package
  - Stata: xt... sequence
  - Other: HLM, MLWIN, Splus, R, winbugs
- Sister formulation: latent growth curve

# Data

# Usual Walking Speed in WHAS

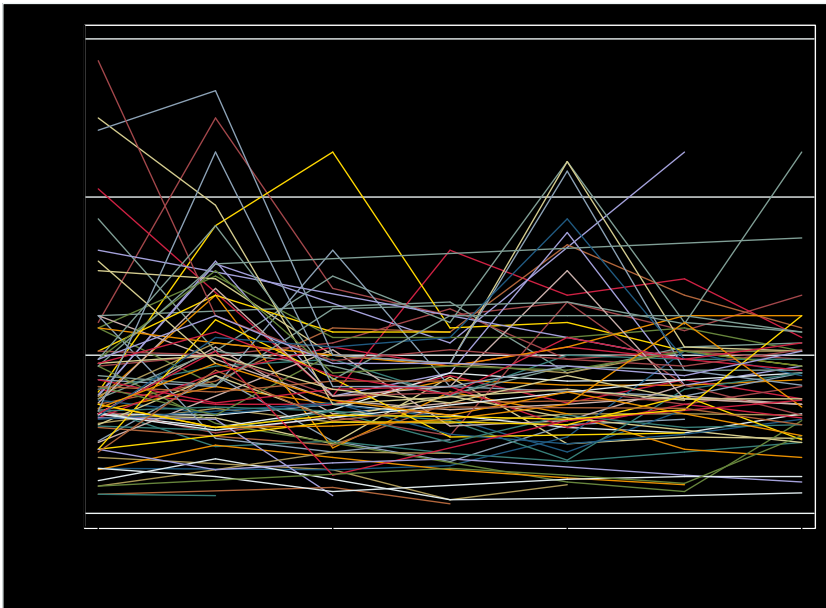
## Panel Plot

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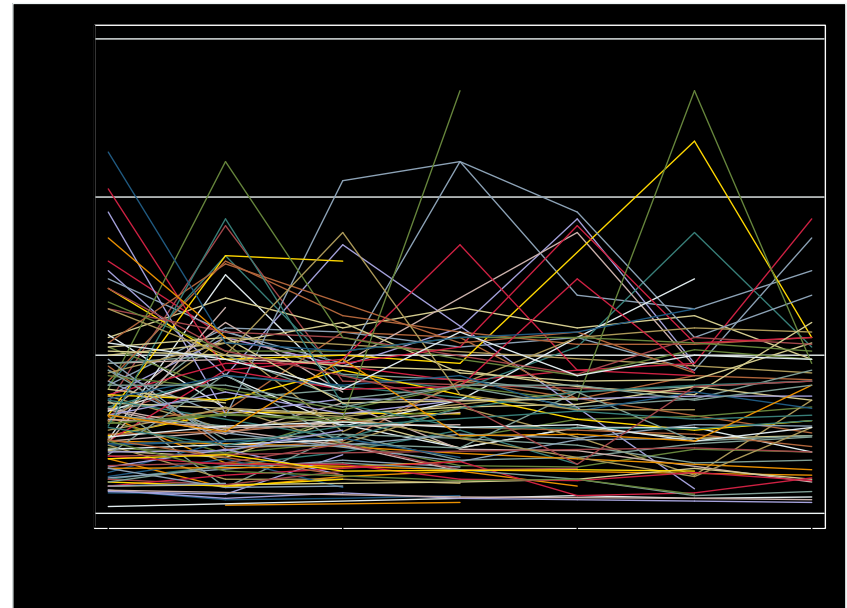


# Usual Walking Speed in WHAS Spaghetti Plots

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Emotionally vital



Emotionally non-vital

# Usual Walking Speed in WHAS

## Does vitality affect walking speed?

Parameter	ML: Independent	GEE: exchangeable	ML: unstructured	ML: Random $b_0$ & $b_1$
Intercept	.58 (.010)	.63 (.035)	.57 (.012)	.58 (.012)
Vitality	.10 (.017)	.075 (.050)	.10 (.020)	.10 (.020)
Time	.0026 (.003)	-.031 (.012)	-.012 (.0022)	-.012 (.002)
Vit*time	-.0015 (.005)	.017 (.018)	.0068 (.0035)	.0062 (.0034)
<b>Main effects model: Intercept, vitality results very similar to above</b>				
Time	.0020 (.0024)	-.0058 (.002)	-.0091 (.002)	-.0094 (.002)

wrong



# Usual Walking Speed in WHAS

## Heterogeneity

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- Residual SD, **variance**: 0.167, **.0280**
  - Represents variability of a woman's speeds "about" her own regression line
- Intercept SD, **variance**: 0.276, **.0762**
  - "Test-retest" estimate =  $.076 / (.076 + .028) = .73$
- Slope SD, **variance**: 0.031, **.00094**
  - 95% of slopes estimated within  $\pm .06$  of  $\sim -.01$
- Intercept, slope covariance: .0020
  - Correlation = .23: better trajectories for better starters
- Unstructured correlations: .6 - >.99
  - Highest late in the study

# Vitality & Walking Speed in WHAS

## Summary

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- Beneficial association with emotional vitality
  - Begin better by  $\sim .1$ ; 95% CI  $\sim [.06, .14]$
  - Moderate evidence: Decline rate  $\sim$  halved
- Remarkable stability evidenced
  - Modest average decline
  - Heterogeneity: moderate  $\downarrow$  to modest  $\uparrow$
  - Stability increased with duration in study
- To advance toward “causation”: much needed
  - Control for confounders
  - Change on change

# Population average v. Subject-Specific

## How to choose?

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- Science
- Advantages of subject-specific models
  - Characterization of heterogeneity—**estimates**
  - May well embody mechanisms
- Advantages of marginal models
  - More robust
    - Standard errors valid if correlation model wrong (GEE)
    - Fixed effect estimates distribution-insensitive
  - Computationally faster, more transportable (GEE)
- An MLE advantage: Missing data treatment

# Why LDA?

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- Top four reasons

4. To inform public policy

- Changes in disability prevalence over time

3. To study natural histories

- Functional trajectories and their etiologies

2. To make prognoses, incorporating history

- Cognitive status transitions

1. To progress from “association” toward “cause”

- Intervention A or risk adoption B changes outcomes

# Transition Models

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- Basic idea: control model for current outcome on all past outcomes
  - “Autoregressive” errors
  - Modify marginal model to include past “Y”s as predictors in model for  $Y_{it}$
- Often assumed: current outcome only depends on the one most immediately past
  - Model for  $Y_{it}$  includes  $Y_{it-1}$  but no other Ys
  - “First order Markov”

*Beckett et al., 1996*

# Some important LDA Challenges

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- Feedback, endogeneity
  - Decline in speed may erode emotional vitality... or, the vital may try harder at the measured walk test
- Dropout, missing data
  - Key distinction: ignorable, non-ignorable
- Nonlinear & clustered trajectories
  - Thresholds, changepoints, trajectory classes

# Take home points

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- If you're out to save **Millions at a Time**©
  - Population average (marginal) model
    - Choice 1: GEE (corr-**robust**) vs. MLE (missing-**robust**)
    - Choice 2: Association structure to fit?
  - Mean trajectory estimates **not sensitive**
- If **one** at a time, or seeking to **target**
  - Subject-specific (random effect) model
  - Benefit if model correct: **heterogeneity** characterization, missing-robust, MLE: precise
- Prognosis based on history: transitions

# An Introduction to Latent Variable Models

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# **LATENT VARIABLES: TRUTH, LIES, AND EVERYTHING BETWEEN**

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Johns Hopkins University**

**December 11, 2007**

# Objectives

For you to leave here knowing...

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- What is a latent variable?
- What are some common latent variable models?
- What is the role of assumptions in latent variable models?
- Why should I consider using—or decide against using—latent variable models?

# ALATENT@

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- 1. Present or potential but not evident or active: latent talent.*
- 2. Pathology. In a dormant or hidden stage: a latent infection.*
- 3. Biology. Undeveloped but capable of normal growth under the proper conditions: a latent bud.*
- 4. Psychology. Present and accessible in the unconscious mind but not consciously expressed.*

The American Heritage Dictionary of the English Language, Fourth Edition, 2000

*Existing in hidden or dormant form but usually capable of being brought to light@*

Merriam-Webster's Dictionary of Law, 1996

# LATENT

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*A. concepts in their purest form...unobserved=or unmeasured=.  
hypothetical*

Bollen KA, Structural Equations with Latent Variables p. 11, 1989

*A. in principle or practice, cannot be observed*

Bartholomew DJ, The Statistical Approach to Social Measurement, p. 12, 1996

*Underlying: not directly measurable. Existing in hidden form but  
usually capable of being measured indirectly by observables*

Bandeen-Roche K, Synthesis, 2006

# LATENT VARIABLES

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Ordinary linear regression model:

$Y_i = \text{outcome (measured)}$

$\underline{X}_i = \text{covariate vector (measured)}$

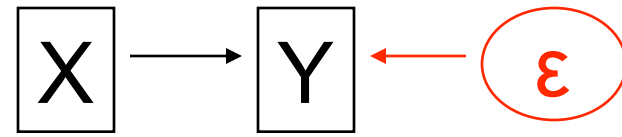
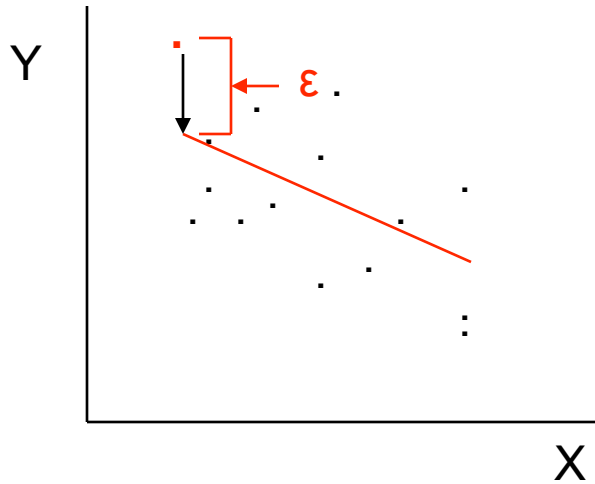
$\varepsilon_i = \text{residual (unobserved)}$

$$Y_i = \underline{X}_i^T \underline{\beta} + \varepsilon_i$$

# Ordinary Linear Regression

## Residual as Latent Variable

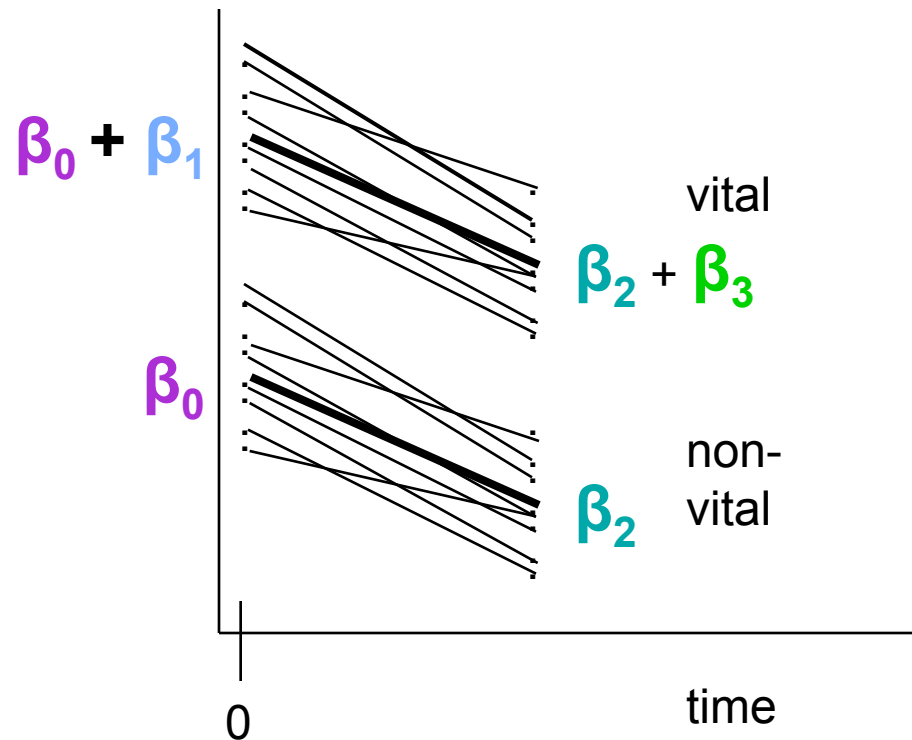
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# Mixed effect / Multi-level models

## Random effects as Latent Variables

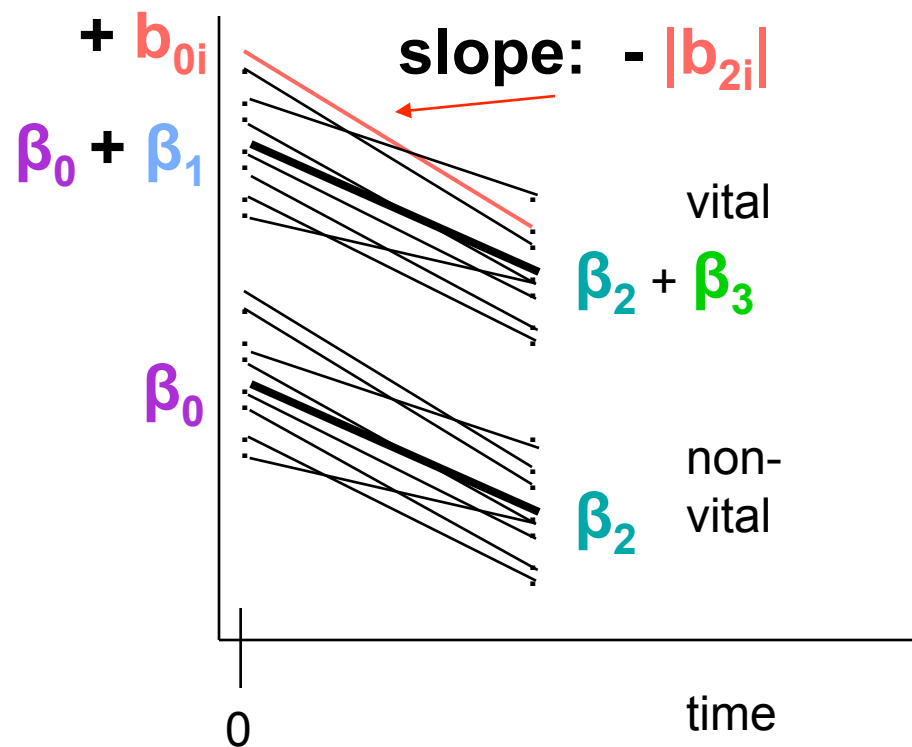
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$$Y_{ij} = \beta_0 + \beta_1 x_i + \beta_2 t_{ij} + \beta_3 x_i \cdot t_{ij} + e_{ij}$$

# Mixed effect / Multi-level models

## Random effects as Latent Variables



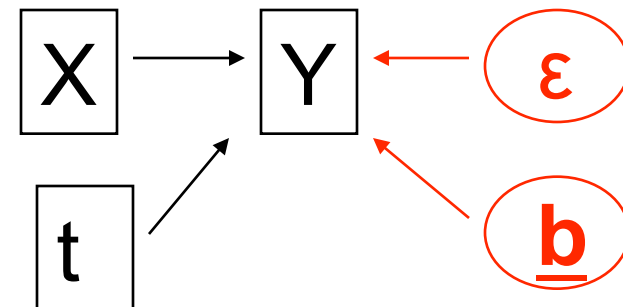
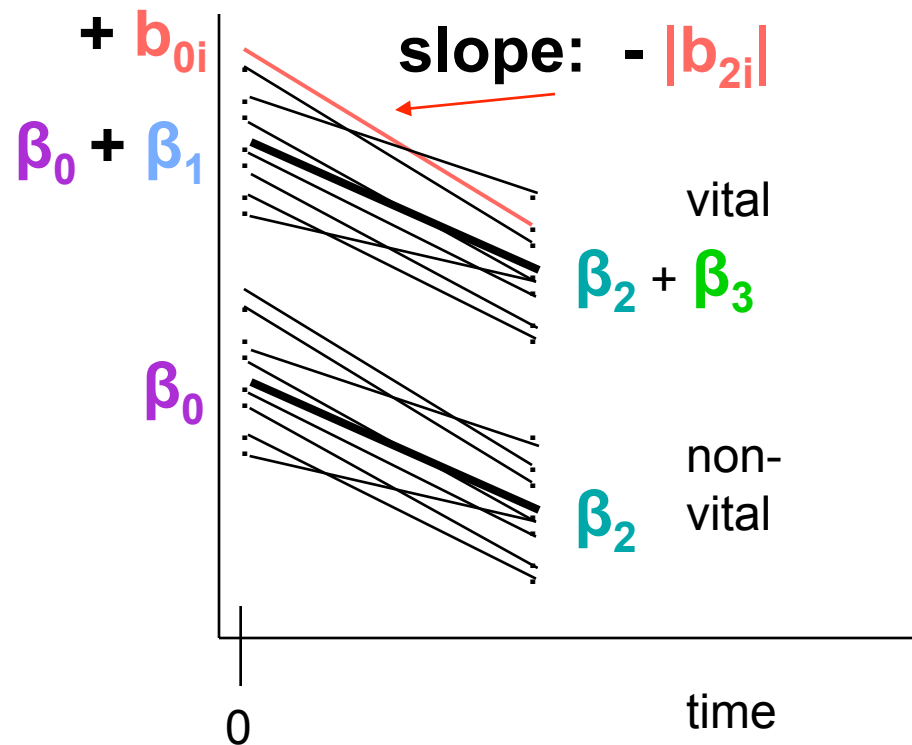
- $b_{0i}$  = random intercept  
 $b_{2i}$  = random slope  
(could define more)
- Population heterogeneity captured by spread in intercepts, slopes

$$Y_{ij} = \beta_0 + b_{0i} + \beta_1 x_i + \beta_2 t_{ij} + b_{2i} t_{ij} + \beta_3 x_i \cdot t_{ij} + e_{ij}$$



# Mixed effect / Multi-level models

## Random effects as Latent Variables

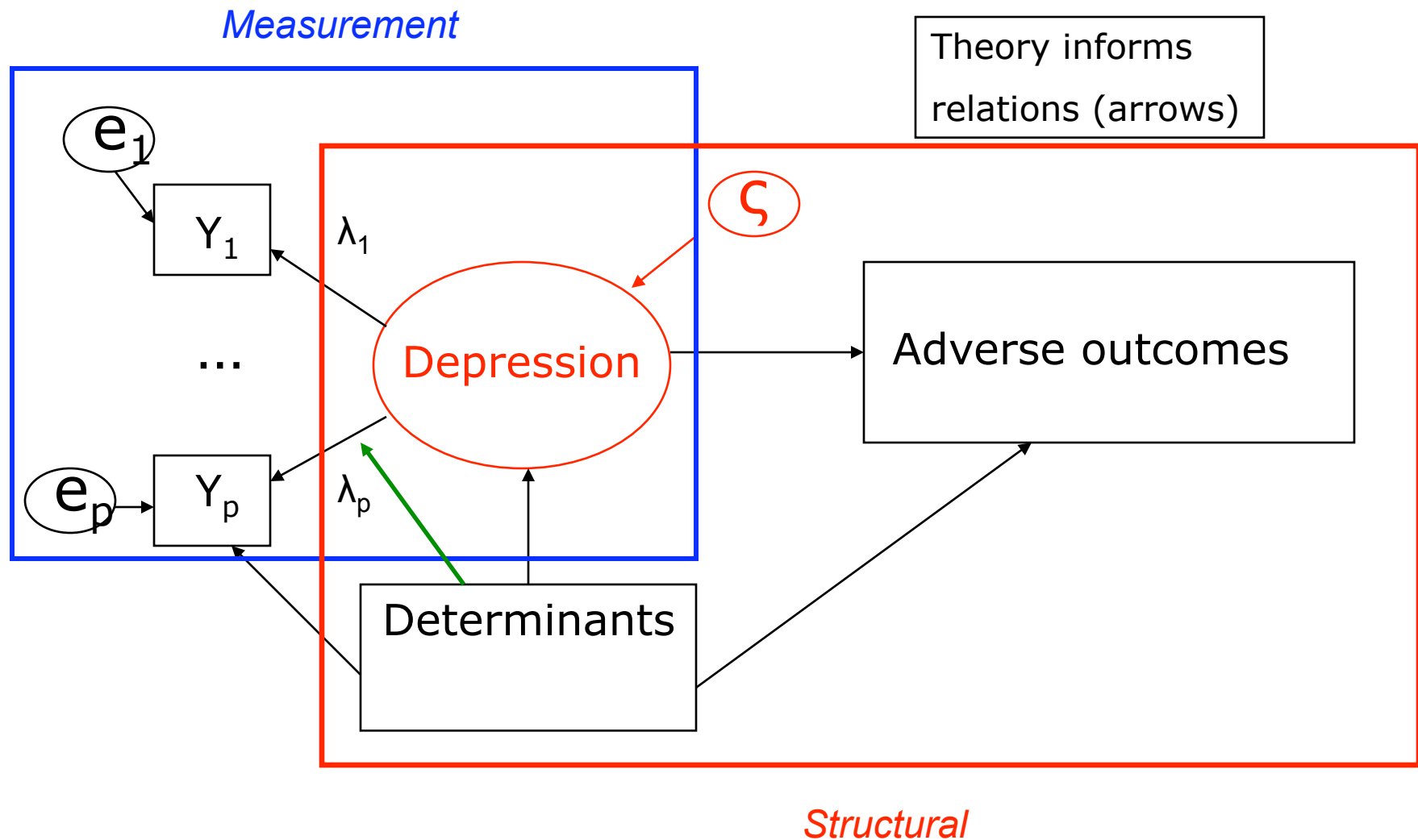


$$Y_{ij} = \beta_0 + b_{0i} + \beta_1 x_i + \beta_2 t_{ij} + b_{2i} t_{ij} + \beta_3 x_i \cdot t_{ij} + e_{ij}$$

# Depression

## Latent Variable Illustration

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# Why do people use latent variable models?

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- The complexity of my problem demands it
  - NIH wants me to be sophisticated
  - Reveal underlying truth (e.g. “discover” latent types)
- Operationalize and test theory
  - Sensitivity analyses
  - Acknowledge, study issues with measurement; correct attenuation; etc.

# Well-used latent variable models

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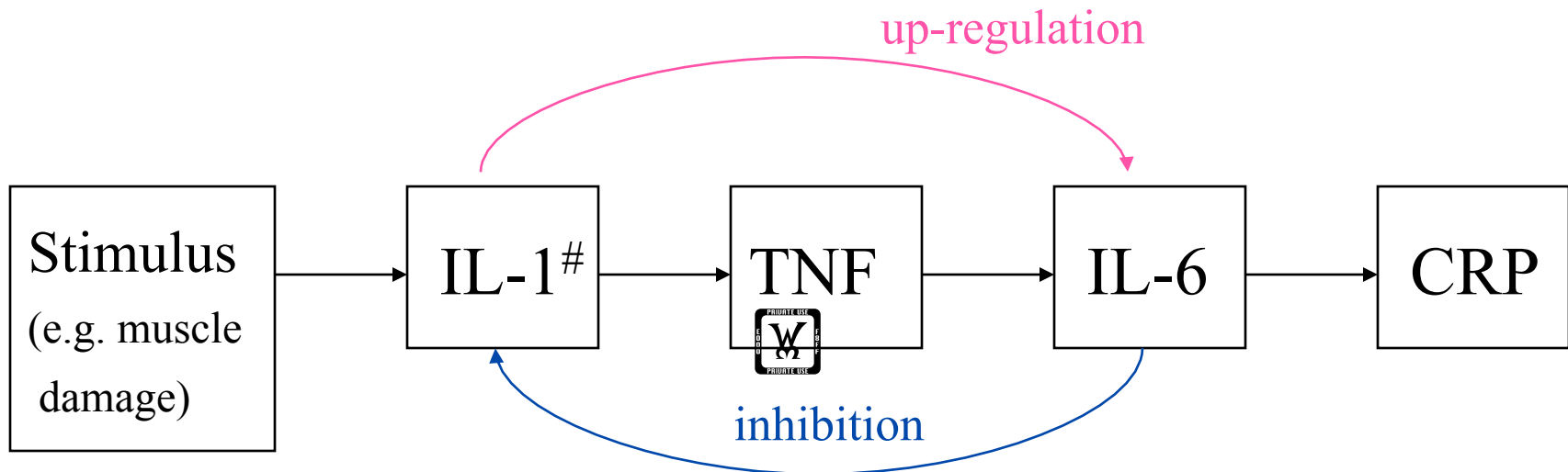
Latent variable scale	Observed variable scale	
	Continuous	Discrete
Continuous	Factor analysis LISREL	Discrete FA IRT (item response)
Discrete	Latent profile Growth mixture	Latent class analysis, regression

General software: MPlus, Latent Gold, WinBugs (Bayesian), NLMIXED (SAS)

LISREL software: LISREL, AMOS, CALIS (SAS)

# Example: Theory Infusion

- Inflammation: central in cellular repair
- Hypothesis: dysregulation=key in accel. aging
  - Muscle wasting (*Ferrucci et al., JAGS 50:1947-54;*  
*Cappola et al, J Clin Endocrinol Metab 88:2019-25*)
  - Receptor inhibition: erythropoietin production / anemia (*Ershler, JAGS 51:S18-21*)



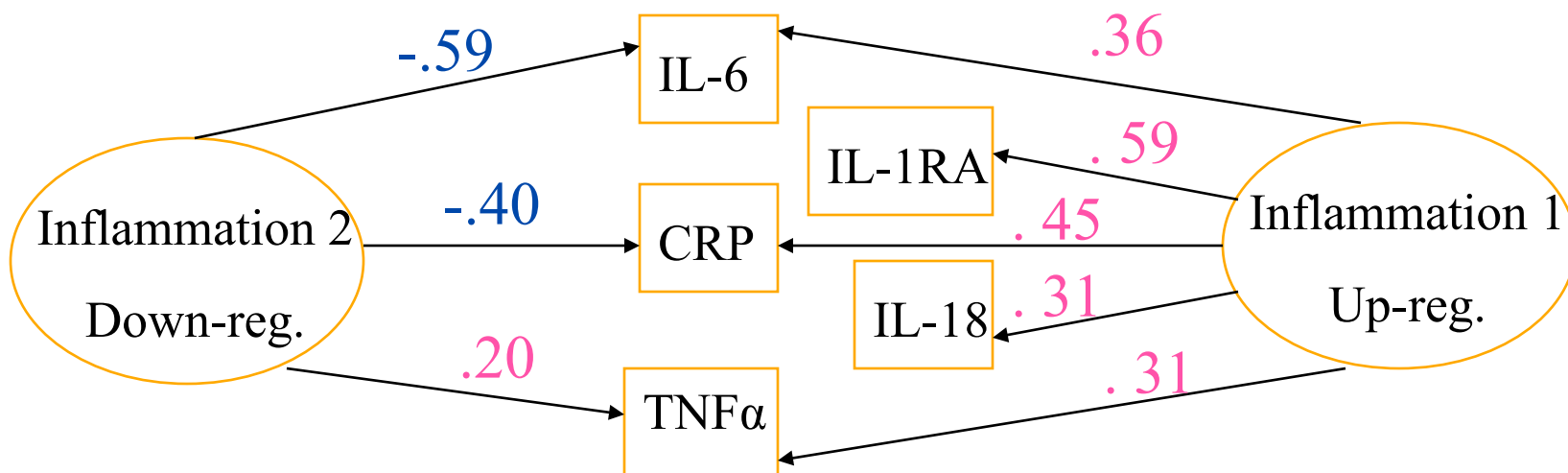
# Difficult to measure. IL-1RA = proxy

# Theory infusion

InCHIANTI data (*Ferrucci et al., JAGS, 48:1618-25*)

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- LV method: factor analysis model
  - two independent underlying variables
  - down-regulation IL-1RA path=0
  - conditional independence



## Application: Post-traumatic Stress Disorder Ascertainment

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### ! PTSD

C Follows a qualifying traumatic event

> *This study:* personal assault, other personal injury/ trauma  
trauma to loved one, sudden death of loved one  
= A<sub>x</sub>@along with gender

C Criterion endorsement of symptoms related to the event □ diagnosis

> Binary report on 17 symptoms = A<sub>y</sub>@

### ! A recent study (Chilcoat & Breslau, *Arch Gen Psych*, 1998)

C Telephone interview in metropolitan Detroit

C n=1827 with a qualifying event

C Analytic issues

> Nosology

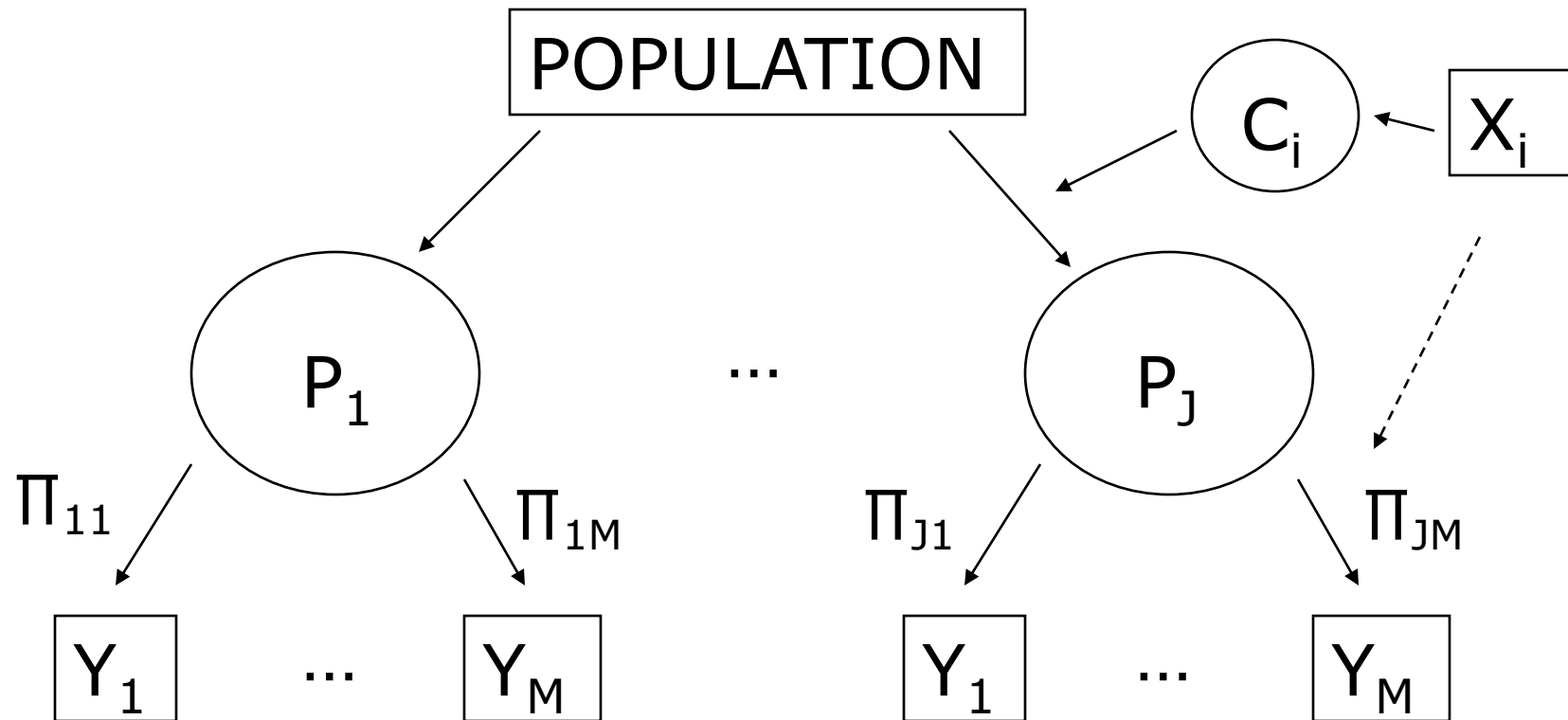
> Does diagnosis differ by trauma type or gender?

> *Are female assault victims particularly at risk?*

# Analysis of underlying subpopulations

## Latent class analysis / regression

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# Analysis of underlying subpopulations

## Method: Latent class analysis/ regression

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- Seeks homogeneous subpopulations
  - **Assumption:** *reporting heterogeneity unrelated to measured or unmeasured characteristics*
  - *conditional independence*, *non differential measurement* by covariates of responses within latent groups : **partially determine features**
- Features that characterize latent groups
  - **Prevalence** in overall population
  - **Proportion** reporting each symptom
  - **Number** of them

## PTSD Study: Descriptive Statistics

Gender	Trauma Type: percentage distribution				n
	<i>Personal Assault</i>	<i>Other Injury</i>	<i>Trauma to loved one</i>	<i>Sudden death</i>	
<i>Male</i>	14.2	<b>37.7</b>	26.9	21.3	964
<i>Female</i>	14.3	26.3	<b>32.2</b>	<b>27.2</b>	863
<b>Total</b>	14.2	32.3	29.4	24.1	1827

! PTSD symptom criteria met: 11.8% (n=215)

C By gender: 8.3% of men, 15.6% of women

C By trauma: *assault (26.9%), sudden death (14.8%), other injury (8.1%), trauma to loved one (6.0%)*

C Interactions: female x assault ( $\square$ ), female x other ( $\square$ )

C Criterion issue? 60% reported symptoms short of diagnosis

## Latent Class Model for PTSD: 9 items

SYMPTOM CLASS	SYMPTOM (prevalence)	SYMPTOM PROBABILITY ( $\pi$ )		
		Class 1 - NO PTSD	Class 2 - SOME SYMPTOMS	Class 3 - PTSD
<b>RE-EXPERIENCE</b>	Recurrent thoughts (.49)	.20	<b>.74</b>	.96
	Distress to event cues (.42)	.12	<b>.68</b>	.88
	Reactivity to cues (.31)	.05	<b>.51</b>	.77
<b>AVOIDANCE/NUMBING</b>	Avoid related thoughts (.28)	.08	.37	.75
	Avoid activities (.24)	.05	.34	.66
	Detachment (.15)	<b>.01</b>	<b>.14</b>	<b>.64</b>
<b>INCREASED AROUSAL</b>	Difficulty sleeping (.19)	.02	.18	.78
	Irritability (.21)	.02	.22	.83
	Difficulty concentrating (.25)	.03	.30	.89
<b>MEAN PREVALENCE-BASELINE</b>		.52	.33	.14

[Omitted: nightmares, flashback; **amnesia**, **interest**, **affect**, **short future**; hypervigilance, startle]

## PTSD: DIAGNOSIS, LCR MEASUREMENT MODEL

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! Method: Regress item responses on covariates **A**controlling@for class  
**C** For simplicity: non-assaultive traumas merged into **A**other trauma@

Variable	Odds Ratio or Interaction Ratio (CI)	By-item Odds Ratio MODEL 2
Female	1.07 (0.93,1.22)	1.07 (0.93,1.22)
Trauma =other than assault (recur.)	<b>3.19 (1.89,5.40)</b>	<b>3.19 (1.89,5.40)</b>
Cue distress x other trauma	<b>0.18 (0.09,0.38)</b>	<b>0.58 (0.36,0.92)</b>
Cue reactivity x other trauma	<b>0.14 (0.07,0.28)</b>	<b>0.44 (0.27,0.72)</b>
Avoid thoughts x other trauma	<b>0.21 (0.11,0.41)</b>	<b>0.68 (0.44,1.05)</b>
Avoid activities x other trauma	<b>0.11 (0.05,0.22)</b>	<b>0.35 (0.21,0.58)</b>
Detachment x other trauma	<b>0.27 (0.13,0.58)</b>	0.88 (0.51,1.49)
Difficulty sleep x other trauma	<b>0.43 (0.21,0.90)</b>	1.37 (0.78,2.42)
Irritability x other trauma	<b>0.28 (0.13,0.61)</b>	0.91 (0.52,1.59)
Concentration x other trauma	0.73 (0.36,1.47)	<b>2.33 (1.35,4.03)</b>

## Summary

### PTSD Analysis

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! The analysis hypothesizes that PTSD is

C a syndrome comprising unaffected, subclinically affected, and diseased subpopulations of those suffering traumas

C reported homogeneously within subpopulations

! The hypotheses are consistent with current diagnostic criteria

! Gender x type interactions: are strongly indicated

C Female assault victims at particular risk

C ... given the subpopulations defined by the model

## Summary

### PTSD Analysis

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! Symptoms appeared differentially sensitive to different traumas

Within classes: those who had a non-assaultive trauma were

C **less prone** to report distress to cues, reactivity to cues, avoiding thoughts, & avoiding activities

C **more prone** to report recurrent thoughts & difficulty concentrating

! Concern: Current criteria may better detect psychiatric sequelae to assault than to traumas other than assault

# Objectives

For you to leave here knowing...

---

- What is a latent variable?
- What are some common latent variable models?
- What is the role of assumptions in latent variable models?
- Why should I consider using—or decide against using—latent variable models?

# DISCUSSION

## The Debate over Latent Variable Models

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! **In favor:** they

- C acknowledge **measurement problems:** errors, differential reporting
- C **summarize** multiple measures **parsimoniously**
- C operationalize **theory**
- C describe population **heterogeneity**

! **Against:** their

- C **modeling assumptions** may determine scientific conclusions
- C **interpretation** may be ambiguous
  - > Nature of latent variables (*existence*)?
  - > Unique (*identifiability*)?
  - > Comparable fit of very different models (*estimability*)?
  - > Seeing is believing (*can the model be checked*)?



# Some closing thoughts

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- Useful information?
  - Enrichment for reading the literature
  - A sense of what's possible
  - Priming for thinking about study design
- Something to build on
  - Courses
  - Seminars
  - Mentoring